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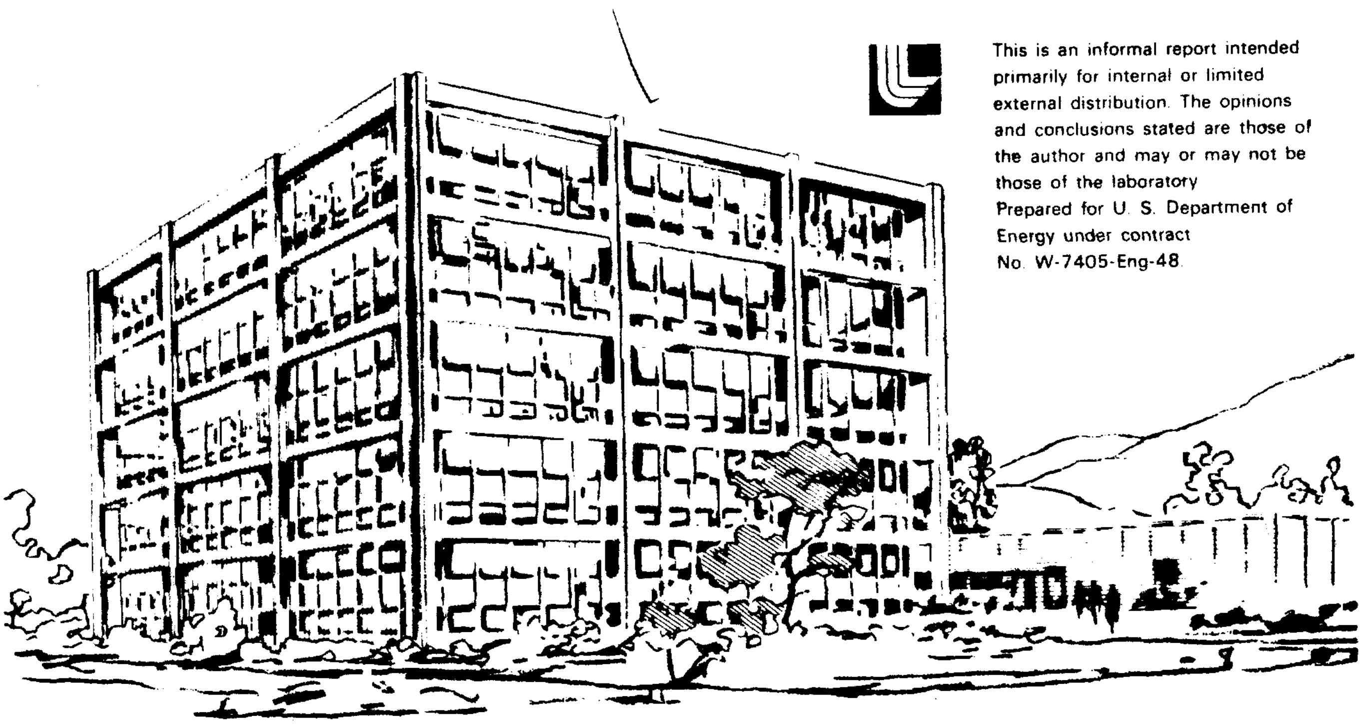

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EFFECTS OF LONGITUDINAL ELECTRIC SELF-FIELD ON THE ACCEPTABLE ENERGY SPREAD IN A FREE ELECTRON LASER*

V. K. Neil

Abstract

In a free electron laser the electron beam density is modulated longitudinally. The longitudinal electric self-field of the particles reduces the stable area in γ - ψ phase space, where γ is the particle's energy in units of the rest energy and ψ is the particle's phase with respect to the wave. A static, self-consistent calculation results in an analytic expression for the stable phase area with the electric self-field included.

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In a free electron laser the action of the wriggler magnetic field and the signal electric field results in a longitrudinal modulation of the charge density in the electron beam. In order to adiabatically trap particles in stable phase, a programmed variation of the wiggler wave number k_w and the wiggler magnetic field B_w has been proposed by P. L. Morton. The purpose of this note is to take into account the longitudinal electric self-field of the electrons in calculating the final stable phase area. The calculation is a static one, and no dynamic effects (such as longitudinal plasma oscillations) are treated.

The notation is Morton's, and a list of symbols is given in Table 1. For a stationary bucket ($\psi_r = 0$) the Hamiltonian may be written as

$$H = \frac{AP^2}{2} - C \cos \psi \ . \tag{1}$$

The value of H corresponding to the separatrix is H = C, and the equation for the separatrix is

$$P_m(\psi) = (2C/A)^{1/2} \ (1 + \cos \psi)^{1/2} \ . \tag{2}$$

Equation (2) does not take into account the reduction in P_m caused by the longitudinal self-field. To calculate P_m, taking this field into account, we must make some assumption regarding the distribution of particles within the bucket. We choose a function $f(P, \psi)$ to describe this distribution. The function f can be a function only of the constants of the motion. The only constant of the motion is $H(P, \psi)$. We define a function G by

$$G = 2(C - H)/A \ . \tag{3}$$

(For purposes of this work A and C are considered constant.) Using Eqs. (1) and (2) we have

$$G = P_m^{\,2}(\psi) - P^2 \,. \tag{4}$$

We choose

$$f(P, \psi) = \frac{2A}{\pi C} \Lambda_o G^{1/2} \tag{5}$$

in which Λ_o is the average charge per unit length of the particles trapped.

The charge per unit length as a function of ψ is given by

$$\Lambda(\psi) = \int_0^{P_m} f(P, \psi)\, dP \,. \tag{6}$$

Inserting Eq. (5) into Eq. (6) and integrating, we find

$$\Lambda(\psi) = \Lambda_o(1 + \cos \psi) \,. \tag{7}$$

The chosen form of f makes our calculation easy, and yet it is physically reasonable.

We now calculate the axial (longitudinal) electric field arising from the trapped particles. We assume, as is quite generally true in a free electron laser (FEL), that $\lambda_s \ll a$, where "a" is the beam radius, and further we asssume that the charge density ρ within the beam varies only slightly with radius. Thus we have

$$\rho(\psi) = \Lambda(\psi)/\pi a^2 \,. \tag{8}$$

The equation satisfied by the axial electric field E_z is (Gaussian units are employed in this work).

$$\nabla^2 E_z - \frac{1}{c^2}\frac{\partial^2 E_z}{\partial t^2} = 4\pi \frac{\partial \rho}{\partial z} + \frac{4\pi}{c^2}\frac{\partial j_z}{\partial t} \,. \tag{9}$$

We neglect the radial dependence of E_z and write

$$E_z = E_0 \sin\psi = E_0 \sin\left[(k_s + k_w)\, z - \omega t\right] . \tag{10}$$

From Eqs. (7) and (8) with $\rho_0 = \Lambda_0/\pi a^2$ we have

$$\rho = \rho_0 \left\{1 + \cos\left[(k_s + k_w)\, z - \omega t\right]\right\} \tag{11a}$$

$$j_z = \rho_0 v_r \tag{11b}$$

Inserting Eqs. (10) and (11) into Eq. (9) we find (with $\beta \equiv v_r/c$)

$$E_0 = 4\pi\rho_0 \frac{[k_s(1 - \beta) + k_w]}{k_w(2k_s + k_w)} . \tag{12}$$

The usual condition for resonance in a FEL is that the resonant particle slip one laser wave-length as it traverses one wiggler period. This condition is equivalent to the relation

$$k_s = \beta k_w/(1 - \beta) . \tag{13}$$

If this relation holds, we have

$$E_0 = \frac{4\pi\rho_0(1 + \beta)}{(2k_s + k_w)} \approx \frac{4\pi\rho_0}{k_s} . \tag{14}$$

In order to modify the Hamiltonian to include the effect of E_z, we add a second term to the expression for $d\gamma/dz$,

$$\frac{d\gamma}{dz} = - C \sin\psi + \frac{eE_z}{mc^2} = \left(\frac{4e\Lambda_0}{k_s^2 a^2\, mc^2} - C\right) \sin\psi , \tag{15}$$

in which we have used Eq. (14) with $\rho_0 = \Lambda_0/\pi a^2$. It remains to define

$$C' = C - (4e\Lambda_0/k_s a^2\, mc^2)\ . \tag{16}$$

Equations (1) and (2) are now valid with the longitudinal self-electric field included in C is replace by C' in these expressions. Our distribution function, Eq. (5), is a self-consistent one if C is replaced by C' in Eqs. (3) and (5).

For a numerical example, we will calculate the value of Λ_0/a^2 that reduces C' from C to C/2. Inserting the expression for C from Table 1 we have

$$\frac{eB_w\ E_s}{2mc^2\ k_w\ \gamma_r} = \frac{8\Lambda_0}{k_s a^2}\ . \tag{17}$$

When capture is accomplished by Morton's method, we have $k_s = k_w{\gamma_r}^2$, and Eq. (17) becomes

$$\frac{eB_w\ E_s\ \gamma_r}{16mc^2} = \frac{\Lambda_0}{a^2}\ . \tag{18}$$

Note that this relation is independent of k_s. We use E_s = 200 kV/cm and B_w = 3.4 kG. The units may be a bit confusing, but it is left as an exercise for the reader to show that, in these units, we have[1]

$$\Lambda_0(\mathrm{kV}) = 30\ I(\mathrm{kA})\ , \tag{19}$$

$$\frac{eB_w}{mc^2} \equiv b_w(\mathrm{cm}^{-1}) = \frac{B_w(\mathrm{kG})}{1.7}\ , \tag{20}$$

[1]In practical units, Eq. (18) becomes $b_w E_s = 4Z_0 J/\gamma_r$, with J the current density and $Z_0 = 120\ \pi\Omega$.

in which $I \equiv \Lambda_0 v_r$ is the average current in the trapped beam (and $v_r \sim c$). With these numbers Eq. (18) becomes

$$1.2I/\gamma_r a^2 = 1 \text{ kA/cm}^2 . \tag{21}$$

For I = 10 kA, γ_r = 100, we have a = 3.5 mm.

If C' = C/2 as in the above example, the value of P_m is reduced by a factor $2^{1/2}$, and the axial length required for one synchrotron oscillation is increased by a factor of $2^{1/2}$. This length determines the length over which the capture can be accomplished in Morton's scheme. From the above example, one would conclude that a beam radius $\sim$1 mm is just too small for 10 kA at 50 MeV. There are other reasons why it is too small, and they will be presented in future notes.

The above steady-state calculation is a first estimate of the effects of the beam's electric self-field. A computer calculation should be performed (taking the self-field into account) to study the capture of particles in stable phase.

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TABLE I

k_w	$\equiv$	$2\pi/\lambda_w$, wiggler wave number
k_s	$\equiv$	$2\pi/\lambda_s$ signal wave number
γ_r	-	energy of resonant particle in units of rest energy
v_r	-	axial speed of resonant particle
E_s	-	signal electric field magnitude
B_w	-	wiggler magnetic field magnitude
e	-	electron charge
m	-	electron rest mass
c	-	speed of light
e_s	-	eE_s/mc^2
b_w	-	eB_w/mc^2
A	-	$2k_w/\gamma_r$
P	-	$\gamma - \gamma_r$
C	-	$e_s b_w/2k_w\gamma_r$
ψ_r	-	phase of resonant particle (zero in this work).

VKN:lab 121v/9v

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